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Publication date:
2009

Document version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
Rørdam, K. B., & Bech, M. L. (2009). *The Topology of Danish Interbank Money Flows*. Finance Research Unit, Department of Economics, University of Copenhagen.

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No. 2009/01

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The Topology of Danish Interbank Money Flows[1]

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Version: December 2008

This paper presents the first topological analysis of Danish money market flows. We analyze the structure of two networks with different types of transactions. The first network is the money market network, which is driven by banks' behaviour on the interbank market, the second is the network of customer driven transactions, which is driven by banks' customers' transactions demand. We show that the structure of these networks differ.

This paper adds to the new and growing literature on network topological analysis of payment systems.

Key Words: Network, Topology, Payment System, Money Market.

1. INTRODUCTION

The recent financial turmoil has highlighted the central role played by the interbank money markets for the smooth functioning of the financial system and implementation of monetary policy. Liquidity evaporated from many parts of the interbank money market and central banks have intervened in force and has de facto replaced private intermediation with public intermediation.

Thus, understanding the inner workings of the money market is of paramount importance in terms of analyzing and responding to financial turmoil.

Theoretical contributions have discussed whether a complete financial structure, where all banks have cross-holdings on each other, or an incomplete structure, where banks only keep the cross-holdings needed, is optimal for hindering contagion from arising, cf. Allen and Gale (2000), Freixas and Parigi (1998) and Freixas et al. (2000). Basically, this is a choice between liquidity saving (banks can keep smaller liquidity reserves if they can raise liquidity via the interbank market) and contagion risk (banks become fragile towards disturbances - in other banks or the network as a whole - if they use the interbank market). In theoretical models, central banks are assumed to make optimal interventions in the interbank market whereby they

[1] The views expressed in this paper do not necessarily reflect those of Danmarks Nationalbank, the Federal Reserve Bank of New York or the Federal Reserve System.

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can hinder contagion from arising, cf. Freixas (2000). But the risk of contagion effects and central banks' possible actions depend crucially on the actual structures on the interbank market.

The large-value payments system is in general the settlement platform for the interbank money market. The lion share of the money market transactions are settled on this platform. Therefore, disruptions in the large-value payment systems can in and by themselves create dislocations in the money market. Moreover, disruptions for a single bank can affect all other banks in the network. Thus, resiliency is crucial. Besides the size of interbank exposures on the money market, the risk of contagion effects also depends on the size of banks and these banks' locations in a network, cf. Lublóy (2006) and Upper and Worms (2004).

Network topology provides a frame work for analyzing the inner working of interbank money flows. During the last couple of years, the physical theory of networks has developed rapidly as it has been shown that many physical networks have many characteristics in common. That is, payment systems have many things in common with other physical networks like the internet or networks for electricity or water supply. In recent years, a new and growing literature on the functioning of payments systems has emerged using the network topological approach. This has led to important new insights into the functioning of financial networks in the US, Japan, Austria and Hungary among others, cf. Soramäki et al. (2007), Inaoka et al. (2004), Boss et. al. (2004), and Lublóy (2006).

Data from the transaction journal of the Danish large-value payment system are used to analyze two economically different networks of interbank money flows. The first network consists of money market transactions, the second of all other transactions. That is, the primary transactions in the payments network are banks' proprietary transactions and customer driven transactions. In contrast to this, the money market network consists of overnight money market loans.

We find that the structure of these networks differ considerably. In the payments network, two commercial banks are responsible for a rather large share of the total activity, whereas there are several major banks in the money market. Both networks are rather concentrated as 10 banks are responsible for most of the transactions in both networks. Seasonal effects are important for the size of the networks. The payments network extends by the turn of the month and quarter and on the first business day following a holiday. In contrast to this, weekday effects drive the calendar effects observed in the money market. Event studies of an operational disruption do not indicate any troubles with regard to the workings of the large-value payment system, whereas payments disruptions by a major participant change the structure of the networks and the level of their activities.

This paper is organized as follows. In section 2 we describe the data and the algorithm used for dividing the data into money market transactions and other transactions. We analyze the network topologies of these economically different networks, which are labelled money market network respectively payments network. Illustrations of these networks are presented in section 3 and section 4 is devoted to a components analysis of the active banks in each network on daily basis. In section 5, the summary statistics of topological measures for both networks are compared and this lay the foundation for the analysis of seasonal effects in section 6. In section 7 we analyze the permanency of links and nodes, which are of importance for the stability of the networks. The final part of the analysis in section 8 is devoted to event studies of two recent incidents in the Danish large-value payment system. Finally, section 9 concludes.

2. THE DATA SET

We have access to all transactions originated over the Danish large-value payment system (Kronos) in 2006[3]. The system was open daily from 7.00 a.m. to 3.30 p.m. and 130 banks, including the central bank, were members of the system in 2006.

Banks use large value payment systems to settle obligations on behalf of their customers as well as their own obligations arising from proprietary operations. An important component of the latter is its overnight money market activities. We use an algorithm similarly to Furfine (1999) in order to separate out from the transaction data set the deliveries and returns of overnight money market loans. We refer to all other transactions as payments.

The algorithm defines a transaction as an overnight money market loan if there is a transaction from bank A to bank B on day t and a reverse transaction from B to A on the same amount plus interest on the following day. The details of the algorithm are explained in the appendix.

A couple of caveats are appropriated as the algorithm's selection criteria do not select overnight money market transactions perfectly. First, the algorithm can only capture overnight loans transferred via the payment system. Second, we can only observe the settlement time of the transactions but not the actual point in time where a bank enter into an agreement on an uncollateralized overnight loan with another bank. An uncollateralized money market loan can be agreed upon earlier in the day of settlement or on previous days[4]. Third, the algorithm does not identify term loans. However, this market is small in Denmark as more than 90 % of the banks lending in the money market for deposits have maturity less than 7 days[5]. Fourth, the borrower and lender registered by the payment system may not be the final ones due to correspondent banking. Despite these drawbacks the algorithm has been used on similar Danish data by Amundsen and Arnt (2005). Thus we will adopt this algorithm and analyze the network topology for the money market on the available data.

We identify two economically different networks by the algorithm's division of our data

1. money market network, which consists of overnight money market loans
2. payments network in which the settlement of customer driven transactions and banks' proprietary transactions take place[6].

The basic characteristics for the money market network and the payments network are shown in table 1 along with results for the full data set.

[3] We exclude transfers to and from auxiliary systems such as the Continuous Linked Settlement for FX trades, CLS, the Danish automated clearing house (Sumclearing) and the Danish central securities depository (VP). The purpose, value and timing of these settlements differ fundamentally from bank to bank transfers.

[4] Tomorrow-next and spot-next trades, which also imply pairs of transactions between two banks on two consecutive days, are agreed upon 1 respectively 2 days before the settlements of the trades.

[5] This is calculated from data on turnover and interest rates in the Danish market for uncollateralized overnight money market lending. In 2006, 12 banks reported these data to the Danish central bank. The central bank estimates an average tomorrow-next interest rate, which is published daily to the market. See Damm and Pedersen (1997) for a detailed description.

[6] All transactions to/from the central bank are in this network since the central bank does not engage in unsecured overnight lending.

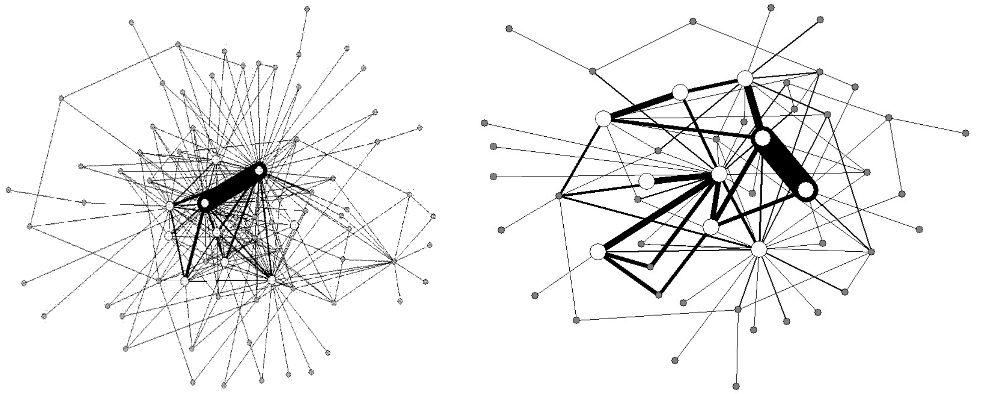
Table 1 Characteristics of the networks, totals for 2006

	Transactions	Money market	Payments
Active banks	130	70	130
Volume of transactions (thousands)	602.7	28.2	574.6
Value of transactions (trillion DKK)	33.3	6.5	26.8
Mean value of transactions (million DKK)	55.3	230.7	46.7
Volume of transactions (per cent)	100.0	4.7	95.3
Value of transactions (per cent)	100.0	19.5	80.5
10 largest banks' share of			
- Volume of transactions	87.3	53.7	88.9
- Value of transactions	91.1	83.0	93.1

Note: Transactions denotes the results for the full data set. Outgoing volume and value from the banks are used to estimate the shares reported.

For each of the business days in 2006 we construct a money market network and a payments network and we use these to obtain aggregated annual results. Each network consists of a number of nodes and links. The banks are nodes and the transactions form links between banks. Two banks are said to be linked if there is at least one transaction between them. Links are directed and the direction follows the flow of money, i.e. from lender to borrower and from payer to payee. If there are more transactions via the same link, the transactions in a network are weighted. The weights are the sum of either value or the number of transactions between two banks.

Figure 1: Payments and money market networks



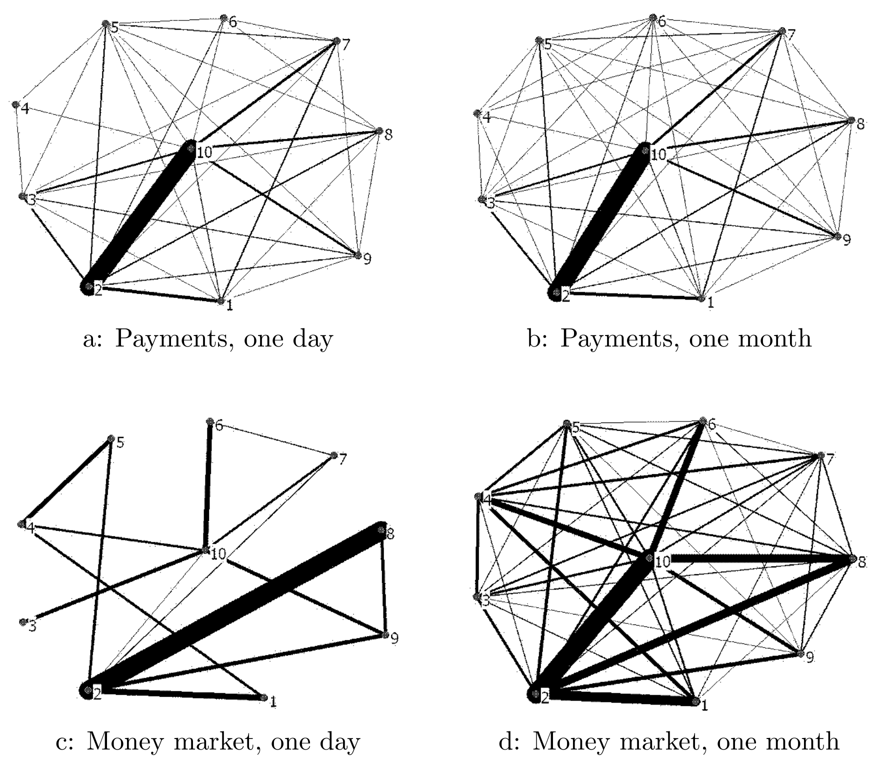
a: Payments   b: Money market

Note: The top-10 banks (large and white coloured) are identified from total value of outgoing payments in 2006. Links are weighted by value.

3. ILLUSTRATION OF THE NETWORKS

The payments and money market networks for a single day in 2006 are illustrated in figure 1. The links are undirected. The thickness of the links is scaled by the value transferred across and the ten banks, which transferred the most money in either network, are highlighted by larger white nodes. Three structural features are immediately obvious. First, more banks are active in the payments network than in the money market network. Second, two large commercial banks play a major role in both networks, but somewhat surprising the important bank-pair in the payments network *is different from* the major bank-pair in the money market. Third, the top-10 banks account for a significant share of the turnover in terms of values in both networks (83.0 respectively 93.1 per cent), which is quite natural as large banks tend to be more connected than other banks. However the top-10 banks' market share in terms of volume is 53.7 per cent in terms of the number of loans in the money market network, cf. table 1. This reflects that the average loan size of the top-10 banks is substantially larger than for other banks in the money market (the average loan size for top-10 banks is 356.6 million DKK and 84.5 million DKK for other banks).

Figure 2: Graphical illustration of the centre of the networks (measured in value)



a: Payments, one day

b: Payments, one month

c: Money market, one day

d: Money market, one month

Note: Data for total payments between the ten largest banks in March 2006 used for one-month-figures. Since the weighting of links in each network depends on the total value of transactions in each network, the thickness of the links is not comparable between networks. The centre of each network consists of the 10 largest banks measured by the total outgoing value of transactions. The top-10 banks are all commercial banks and bank 2-10 are the same in both networks, whereas bank 1 differ between the payments network and the money market network.

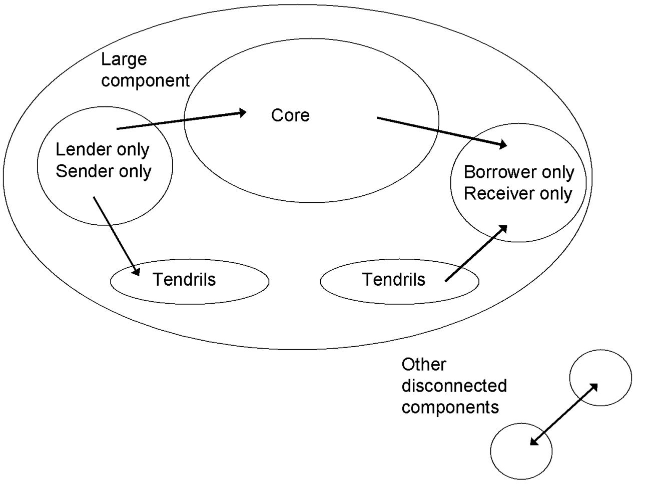
In order to better understand the structure of flows among large banks we plot the network of only the ten largest banks in figure 2. We do so in two ways. In the first column of figure 2 we show networks based on transactions for one day whereas the second column show the networks based on transactions for an entire month. The structural differences between the payments and the money market networks are striking. The one day centre of the payments network is almost complete[7] whereas the degree of completeness is 20.0 per cent on average in the centre of the one day money market.

4. COMPONENTS

Nodes in a network can be divided into groups depending on how they connect to other nodes. A network is comprised by a set of disconnected components within which nodes are linked by an undirected path and do not have links to nodes outside the component. Many empirical investigations find that one of the disconnected components is several orders of magnitude larger than the other disconnected components, cf. Dorogovtsev and Mendes (2002), Albert and Barabási (2002), Soramäki et al. (2007). In contrast, we find that the payment and interbank money market networks consist only of a single component on every day.

We divide the networks into four subcomponents[8], cf. table 2. First, we have the core which consists of banks that are connected to each other via a directed path. Attached to the core are two peripheral set of banks that are on a directed path to or from the core. As such the core facilitates the circulation (or intermediation) of funds within the network whereas banks in the peripheral groups are either senders or receivers of funds only. Finally, a limited number of banks belong to so-called tendrils, which consists of nodes that are on a directed path to or from the peripheral components.

Figure 3: A network and its components



[7] The degree of completeness is at its maximum of 100 per cent in a complete network and at its minimum in a tree network, where the degree of completeness is equal to 1 divided by the number of nodes. Complete and tree networks are stylized networks, which are not observed empirically. See the appendix for an illustration of stylized networks.

[8] In the network topology methodology the large component is known as the *Giant Weakly Connected Component.* The core of the network is denoted the *Giant Strongly Connected Component* and the lender/sender (borrower/receiver) only components as the *Giant In-Component* (*Giant Out-Component).* Finally, the other disconnected components are denoted *Disconnected Components*, cf. Dorogovtsev and Mendes (2002).

Table 2: Components in the networks

	Payments	Money Market
Nodes connected by a directed path	The core	The core
Nodes on a directed path to core/tendril	The sender only	The lender only
Nodes on a directed path from core/tendril	The receiver only	The borrower only
Other nodes	Tendril	Tendril

Note: The lender/sender (borrower/receiver) only component can submit (receive) transactions to (from) either the core of the network or to (from) a tendril.

Table 3: Components of the networks, 2006

Component	Comp.'s shares	Mean	Median	Min	Max	Std	Value Out	Value In	Capital Average
Payments									
	Per cent	Number of nodes					Per cent		Billion DKK
Network	100.0	89.0	89.0	76.0	113.0	5.3	100.0	100.0	23.1
Core	67.7	60.3	60.0	48.0	86.0	6.2	99.6	99.6	33.2
Sender only	16.2	14.4	14.5	3.0	24.0	3.8	0.3	0.0	2.6
Receiver only	15.1	13.5	13.0	4.0	29.0	4.0	0.0	0.3	2.7
Tendrils	0.9	0.9	0.0	0.0	5.0	1.1	0.0	0.0	2.0
Money market									
	Per cent	Number of nodes					Per cent		Billion DKK
Network	100.0	43.6	44.0	32.0	53.0	4.1	100.0	100.0	42.3
Core	62.9	27.4	28.0	3.0	43.0	6.8	93.5	93.3	61.6
Lender only	16.9	7.4	6.0	0.0	24.0	5.1	5.5	0.7	11.0
Borrower only	16.9	7.4	6.0	0.0	27.0	4.9	0.7	5.7	10.9
Tendrils	3.3	1.4	1.0	0.0	24.0	2.3	0.3	0.2	8.3

Note: The components' shares (Comp.'s shares) of the network are calculated from the mean of the number of nodes. The shares of the value are calculated for in- respectively outgoing payments and the last column contains the average level of capital for the banks in each component. The large maximum value of tendrils in the money market occurs on the first business day in 2006.

Our results show that 89.0±5.3 (the mean plus/minus the standard deviation across days) banks are active in the payments network on average in 2006. 60.3±6.2 banks belong to the core, cf. table 2. The money market network is smaller with only 43.6±4.1 banks being active on an average day in 2006. The size of the core in the money market was 27.4±6.8.

In both networks, most of the transactions are transferred within the core, cf. table 3. As measured by capital[9], banks in the core are larger than banks in other components in both networks. As a number of smaller banks are active in the payments network only the average capital level of banks is larger in the money market than in the payments network.

The lion share of value in both networks is transferred within their respective cores. For the payments network the share is 99.6 per cent of the total value,

[9] Banks' capital is their productively employed capital, which comprises deposits, issued bonds, subordinated capital contributions and equity capital. Banks' productively employed capital is used to determine the fixed membership fee of the Danish large-value payment system.

whereas in the money market network it is 93.5 per cent. Banks in the peripheral groups comprise almost all of the remaining value in both networks.

5. SUMMARY STATISTICS FOR THE NETWORK TOPOLOGIES

A detailed analysis of the structural differences between the networks across time is difficult by visualization. Therefore, we consider a set of statistical measures common in the network topological approach in this section[10]. We will focus on statistics of network activity in the core of the networks as the core plays a key role in determining the activity and the well-functioning of a payment systems network due to its intermediary role in distributing liquidity among banks in demand and supply of it, cf. table 3.

5.1. Basic network properties

The turnover in the payments network[11] is larger both in value and volume than in the money market network, cf. table 4. Moreover, as mentioned above the number of active banks within the core of the payments network is larger than in the money market network. That is, the banks in the payments network have more transactions with each other compared with the money market network. The link weight is a measure, which take the value respectively the volume of the transactions, which go via a certain link, into account. The link weight in value is slightly lower in the money market network than in the payments network (322.7 million DKK respectively 374.7 million DKK on average), whereas the link weight in volume is significantly larger in the payments network than in the money market network. This explains the difference in the average size of a transaction in the two networks in table 1.

Figure 2 showed that the centre of the networks for the top-10 banks for the monthly averages were almost complete as the top-10 banks tend to form links with almost all other top-10 banks. But when all links in the networks are considered, the actual number of links formed is substantially smaller than the potential number of links. This reflects that banks in the periphery of a network tend to form fewer links than the banks in the core. For both networks only 1 out of 10 possible links are formed on a given day with a slightly lower connectivity in the payments network (8.3 ± 0.8 per cent) than in the money market network (11.2 ± 5.8 per cent).

Another interesting measure is the reciprocity, which measures the share of links between banks for which there is a link in the opposite direction. The reciprocity is virtually the same in the two networks as 1 of out 4 links have transactions in both directions. The reciprocity in the money market network is substantially larger than in the Fed Funds Market, whereas the reciprocity in the payments network is a bit larger than in Fedwire[12].

[10] A list of the topological measures including a short description of these can be found in the appendix. See Hekmat (2006) for a thorough description of the physical concepts in network topology.

[11] The summary statistics for the payments network are in line with the results for the transactions network (the whole data set) since most of the observations in the transactions network are the same as in the payments network. Statistical measures for the transactions network are reported in table A2 in the appendix.

[12] Overnight loans between banks are borrowed or lend in the Market for Federal Funds (Fed Funds Market) in the US. The Fedwire Funds Service (Fedwire) is a real-time gross settlement system operated by the Federal Reserve System in the US.

The time series pattern over the time period for the turnover in value and volume, the activity in nodes and links and the connectivity and reciprocity for both networks are shown in figure 4. Some of these measures reveal seasonality especially around quarter ends. We discuss seasonal effects further in section 6.

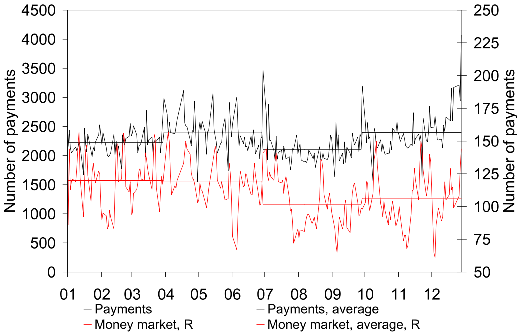
Table 4: Summary statistics, payments and money market networks, 2006

	Mean	Median	Min	Max	Std
Payments					
Volume	2,162.4	2,127.0	1,493.0	3,434.0	283.8
Value	105.5	101.3	29.5	186.9	27.3
Nodes	60.3	60.0	48.0	86.0	6.2
Links	282.6	277.0	202.0	489.0	40.9
Connectivity, per cent	8.3	8.2	6.7	11.2	0.8
Reciprocity, per cent	22.8	22.8	18.0	27.3	1.8
Clustering	0.5	0.5	0.4	0.7	0.1
Average path length	2.5	2.5	2.3	2.7	0.1
Average node degree, k	4.8	4.7	4.0	6.4	0.4
Link weight, value	0.4	0.4	0.1	0.7	0.1
Link weight, volume	7.7	7.7	5.0	10.1	0.8
Node strength, value	1.8	1.7	0.5	3.4	0.5
Node strength, volume	36.7	36.3	24.1	54.6	4.3
Money market					
Volume	86.4	88.0	4.0	144.0	26.0
Value	22.9	22.1	0.3	45.2	8.1
Nodes	27.4	28.0	3.0	43.0	6.8
Links	75.0	76.0	4.0	132.0	23.3
Connectivity, per cent	11.2	10.2	6.7	66.7	5.8
Reciprocity, per cent	26.2	26.4	10.0	50.0	5.5
Clustering	0.2	0.2	0.0	0.5	0.1
Average path length	2.9	2.9	1.3	4.6	0.4
Average node degree, k	2.7	2.7	1.3	3.5	0.3
Link weight, value	0.3	0.3	0.1	1.5	0.1
Link weight, volume	1.2	1.1	1.0	1.8	0.1
Node strength, value	0.9	0.8	0.1	2.0	0.3
Node strength, volume	3.1	3.1	1.3	4.3	0.4

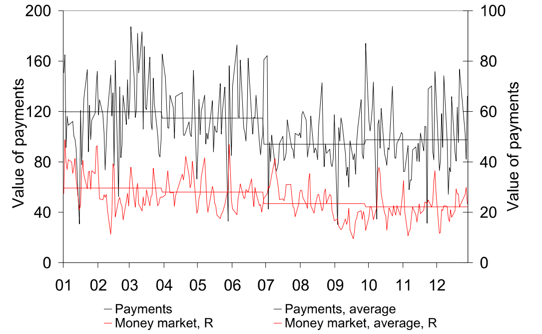
Note: The value and the link weight and node strength in value are in billion DKK. Clustering and the average path length are estimated using outgoing payments from a node. The reported summary statistics refer to the average of the daily observations for the core.

The reciprocity in the Fed Funds Market is 6.5 per cent in Bech and Atalay (2008), whereas the reciprocity in Fedwire is 21.5 per cent in Soramäki et al. (2007).

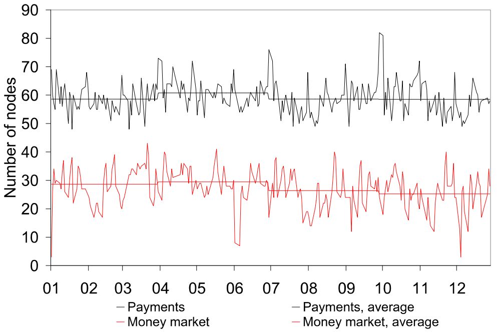
Figure 4: Activity of the payments and money market networks, 2006



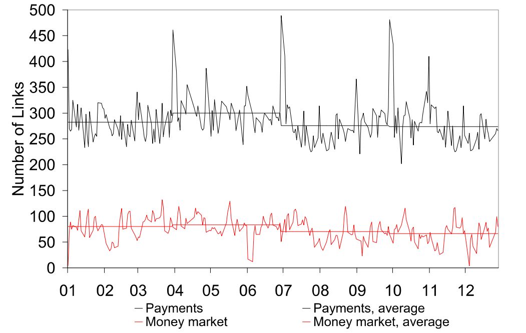
a: Volume



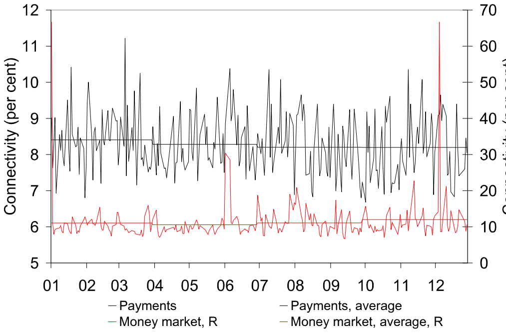
b: Value



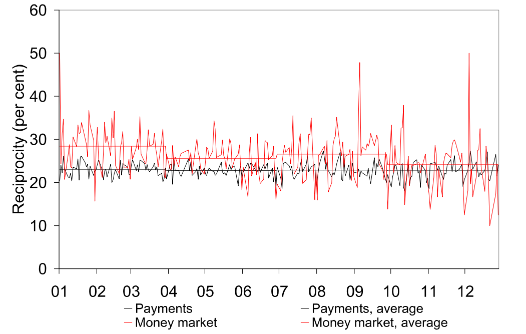
c: Nodes



d: Links



e: Connectivity



f: Reciprocity

Note: All figures are for the core in 2006 and the months are labeled with numbers from 1 to 12. The value of payments (panel b) is in billion DKK. All figures include quarterly averages of the variables. Even though the value is downward sloping and the volume increases during the year in the payments network, the average value of a payment has been almost unchanged in the period 2003-2007.

5.2. Correlations of basic network properties

The different basic network statistics are not independent of each other. For example a larger network might imply more links. Hence we look at the correlation over time between the different network characteristics.

The number of banks active is positively correlated with the number of interactions (links) in both networks. Moreover, more activity in terms of value and volume generate more links and nodes in the networks. Furthermore, the value and volume of the networks are positively correlated. This reflects the patterns in figure 4, where the activity in volume and value tend to covariate with the size of the networks (nodes and links).

The connectivity in the money market is negatively related to any measure of activity (value and volume) and size (nodes and links). In general, the denseness of the money market network (reciprocity) is uncorrelated with any other measure with the possible exception of the slightly positive correlation between reciprocity and connectivity. This reflects that a bank, which become active in the money market, tend to have only a few links to other banks.

In the payments network, connectivity is negatively correlated with the number of active banks and slightly negatively correlated with the number of links, whereas the connectivity is virtually uncorrelated with the activity in the payments network. The reciprocity is negatively correlated with the number of active banks and positively correlated with the connectivity, but almost uncorrelated with the remaining variables. That is, the payments network does not become denser as the activity increases. This result is contrary to Soramäki et al. (2007), which find that the correlation between nodes (links) and connectivity are quite strong and positive in Fedwire.

Table 5: Correlations of basic network properties, 2006

Payments

	Value	Volume	Nodes	Links	Connectivity	Reciprocity
Value		0.58	0.25	0.44	0.14	0.26
Volume			0.50	0.68	-0.02	0.09
Nodes				0.86	-0.72	-0.36
Links					-0.28	-0.17
Connectivity						0.48
Reciprocity						

Money market

	Value	Volume	Nodes	Links	Connectivity	Reciprocity
Value		0.62	0.49	0.55	-0.34	0.04
Volume			0.92	0.98	-0.57	0.09
Nodes				0.95	-0.69	0.05
Links					-0.57	0.07
Connectivity						0.25
Reciprocity						

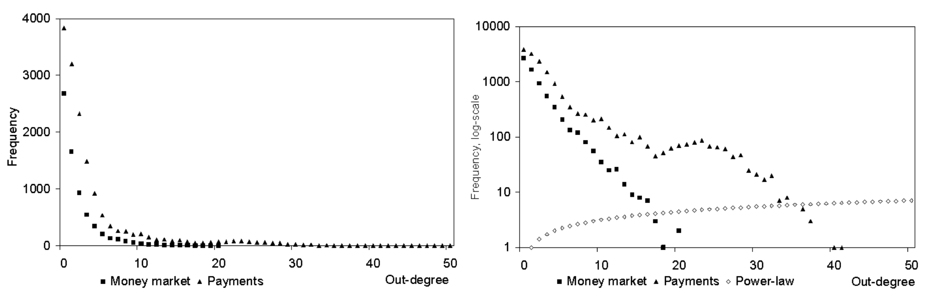
5.3. Degree distribution

An important characteristic of a node in a network is the number of links, which originate from a node and the number of links terminating in a node. The average node degree is equal to the number of links divided by the number of active banks.

In the payments network, the average node degree is 4.8±0.4, which is almost double the average node degree of 2.7±0.3 in the money market network. In the payments network, the maximum number of links originating from (terminating in) an active bank is 29.0±3.9 (34.6±4.4), cf. table A1 in the appendix. In the money market network, the number of links originating from (terminating in) an active bank is 10.3±3.4 (10.3±3.6). That is, banks within the money market tend to have fewer links to other banks than active banks in the payments network.

In both networks, most banks have only a few links that either originate from (or terminate in) these banks. In other words, the distribution of in-degrees and out-degrees are fat-tailed, cf. figure 5a. A number of studies have shown that in- and out-degrees in large-value payment systems in the US, Japan and Austria follow power-laws[13], cf. Inaoka et al. (2004), Soramäki et al. (2007) and Boss et al. (2004). In a random network, the distributions of in- and out-degrees follow a Poisson distribution, cf. Dorogovtsev and Mendes (2002) and Newman (2005). Neither a power-law distribution, nor a Poisson distribution capture the distribution of the in- and out-degrees correctly in the Danish case, cf. figure 5b and 6. In the payments network, the exponential distribution or the negative binomial distribution capture the actual distributions of in- and out-degrees quite well, whereas the exponential distribution is closest to the actual values of in- and out-degrees for the money market network, cf. figure 6.

Figure 5: Distributions of out-degrees

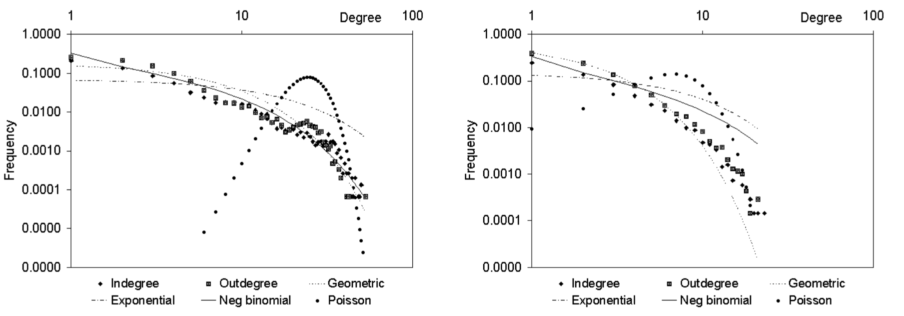


a: Distribution of out-degrees b: Log-scale including power-law

Note: The y-axis is in log-scale in panel (b). The data are for the whole network of interconnected banks. Only out-degrees are shown here, but figures for in-degrees are similar.

[13] A power-law is a distribution for which there is a scale effect, i.e. $P(X = x) \sim x^{-\gamma}$.

Figure 6: Distribution of in- and out-degrees



a: Payments   b: Money market

Note: Both x- and y-axis are logarithmic. The data are for the whole network of interconnected banks. The leftwing tail of the Poisson distribution in panel (a) has been cut off to keep a clear picture. This choice is reasonable since the in- and out-degrees for the payments network are clearly not Poisson distributed.

5.4. Distance measures

The average path length is the average number of links, which connects two banks via the shortest possible path, i.e. the average path length measures across how many links 1 DKK must pass to reach another bank. Our results show an average path length of 2.5±0.1 in the payments network and 2.9±0.4 in the money market, cf. table 4. The corresponding values for Fedwire and the Fed Funds Market are 2.6 respectively 2.7, cf. Soramäki et al. (2007) and Bech and Atalay (2008). The maximum distance between two banks (measured by the number of links) is the diameter, which is 5.5±0.7 for payments network and 6.7±1.3 for the money market network, cf. table A1 in the appendix. This is substantially smaller than the diameter in Fedwire of 6.6 on average and the diameter in the Fed Funds Market of 7.3, cf. Soramäki et al. (2007) and Bech and Atalay (2008).

More than half of the other banks in the payments network can be reached within 2 nodes cf. table A1. Increasing the distance to 3 implies that 91.2±2.7 per cent of the nodes can be reached and by the distance 5 almost all banks are reachable. In a study for the Fedwire, Soramäki et al. (2007, table 3) finds that the mass distribution function reaches almost 100 percent within the distance 4. The larger distance between banks in the money market implies that only 42.1±9.5 (71.6±10.0) per cent of the banks can be reached within a distance of 2 (3).

5.5. Clustering and centrality measures

Clustering measures the degree to which two banks, which are both linked up with bank x, are also linked up with each other. In other words, in the payments network there is a 50 per cent chance that two neighbours of a node are also linked to each other whereas there is only a 1 out of 5 chance in the money market network. In both networks, the clustering coefficient is much higher than the connectivity so neither of the networks is random[14].

[14] In a random network, the clustering coefficient is equal to the connectivity. A random network is constructed by adding links at random to a given set of nodes. This is a stylized type of network, which is unobserved in reality.

An omnipresent question in network theory is the relative importance of different nodes and links usually referred to as centrality. We have already discussed the notion of degree above. The most connected bank on any given day in our sample had 53 outgoing (55 incoming) links for the payments network and 21 outgoing (24 incoming) links for the money market network. Another way to measure importance is node strength which measures the amount (or number) of payments or loans processed by a participant. According to this measure the largest node across all days processed outgoing payments worth 74.2 billion DKK in the payments network and lend out loans worth 21.1 billion DKK in the money market network on any given day. The largest (directed) link between any two banks in the two networks transferred 58.8 billion DKK worth of payments and 12.2 billion DKK worth of loans. In a relative sense the largest node and link in the payments network accounted for 52.9 and 43.7 per cent, respectively, of the total value transferred on any day. In the money market the equivalent "market share" numbers where 71.0 and 64.5 percent, respectively.

Another measure of centrality is betweenness, which is a measure of the number of paths between other nodes that run through node i. The more paths node i handles, the more central is this node in the network. The measure can also be applied for links to identify the most important links between banks. Results in table A1 in the appendix show that the average betweenness for links is almost identical in both networks (29.2±8.4 in the money market and 30.3±3.7 for the payments network), whereas the betweenness for nodes in the money market network is 40 per cent lower than in the payments network, i.e. each node in the money market handles fewer paths than banks in the payment network.

6. SEASONAL EFFECTS

To access the implications of seasonal effects we regress 8 different topological measures on a set of dummies for holidays, weekdays and liquidity provisions by the Danish central bank in addition to the regular liquidity adjustments on Fridays. Results are shown in table A3 and A4 in the appendix.

For the payments network, the effects on the first business day following Danish or US holidays are significant for links, value, volume and average node degree. This network is extended at every turn of month and quarter both considering the number of active banks, links, value, volume and average node degree. Moreover the connectivity decreases significantly by the turn of quarter. These effects are due to large quarterly interest and repayment on mortgage loans, which is the prime source of funds in the Danish housing market, and monthly payments of salaries, social benefits and taxes etc., which initiate more transactions that usual. The network is largest on Fridays (in nodes, links and value) and smaller by the beginning of the week (in nodes, links, volume and average path length). Both planned and unexpected liquidity adjustments increase the number of links and the average node degree significantly.

For the money market there are significant weekday effects for nodes, links and volume, especially on Mondays, Tuesdays and Fridays, i.e. the demand for interbank liquidity decreases on Friday, which is the first days in the weekly liquidity schedule. This affects connectivity positively. The same pattern of a significantly decreasing number of nodes and links, less volume and increased connectivity is observed by the turn of the month. Only the average node degree and the connectivity are positively affected by the turn of quarter. Unexpected liquidity adjustments increase the average node degree and decrease the average path length. In contrast to this, there are no effects from expected liquidity adjustments or from holidays.

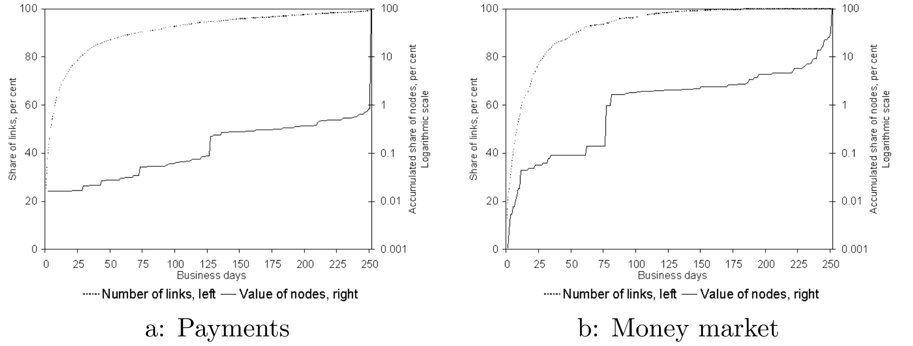
7. PERMANENCY OF NODES AND LINKS

We find that 99.2 per cent of the total value (30.8 per cent of the total volume) is settled by the 40 nodes that are active on each business day in the payments network. Between these nodes there were 26 permanent links, which accounted for 74.6 per cent of the value transferred (77.6 per cent of the volume). Most links only exist for few business days. These occasional[15] links only handle 0.02 per cent of the value and 7.7 per cent of the volume in the payments network. Compared with Hungary, a smaller fraction of the value is transferred via permanent links in Denmark, cf. Lublóy (2006). One reason might be that the Hungarian large-value payment system has larger banks as its members, whereas banks of different size are members of the Danish RTGS-system.

In the money market there is no bank, which is active on all business days. The most permanent link existed for 189 days out of 252 business days and this link handled 19.2 per cent of the total turnover in the money market. The bulk of occasional links is larger than in the payments network. 11.2 per cent of the value (26.7 per cent of the volume) is transferred through occasional links in the money market. 7 banks were active on all business days in 2006 and these handled 10 per cent of the volume and 66.7 per cent of the value in the money market.

Thus in both networks, the frequency of links is skewed to the left and the frequency of nodes[16] is skewed to the right, cf. figure 7.

Figure 7: Frequency of links and nodes



a: Payments b: Money market

Note: The number of links is measured in per cent, whereas the value of nodes is measured in per cent and accumulated. The (accumulated) share of links (nodes) is plotted against the number of business days. The data are for the whole network of interconnected banks.

[15] We consider links existing for less than 26 days as occasional, i.e. these links exist on less than 10 per cent of the business days.

[16] Corrected for the value handled by each node. This reflects that a node, which handles very valuable payments, is more important than a node, which handles less valuable transactions.

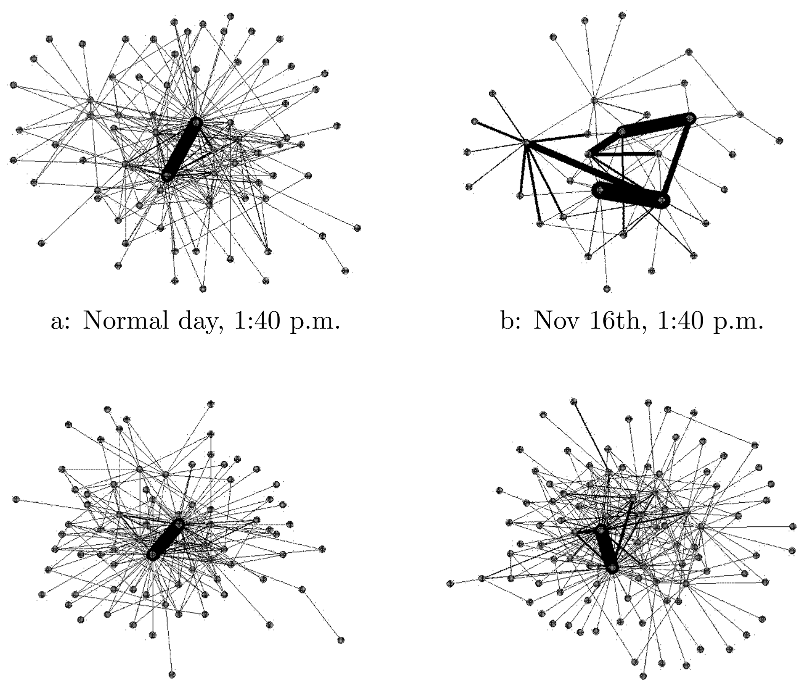
8. EVENT STUDIES

In order to investigate how the networks respond to disturbances we consider two case studies of operational events[17]. The first event is an intraday operational disruption of the Danish large-value payment system; the second is payment disruptions by a major participant on multiple days.

8.1. Operational disruption of the system

On Thursday November 16th 2006 the Danish large-value payment system experienced an intraday operational failure, cf. Danmarks Nationalbank (2007). The system opened as usual, but due to an unsuccessful software update the settlement process stopped after the first few minutes and the system remained down for more than 6 hours. When the system came up again later that day, a large bulk of transactions was settled immediately. As a consequence of this event, the Danish central bank extended the closing of the system with 15 minutes but only two transactions took place after the official closing time at 3:30 p.m. Furthermore, the central bank provided extra liquidity to the market by repurchases of certificates of deposit.

Figure 8: Operational disruption in the payments network



a: Normal day, 1:40 p.m.

b: Nov 16th, 1:40 p.m.

c: Normal day, End of day

d: Nov 16th, End of day

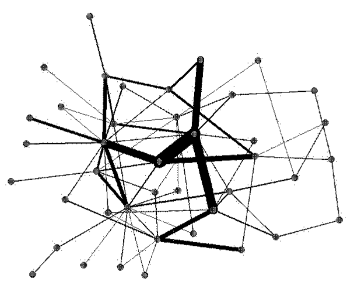
Note: The figures are weighted by value.

[17] Event studies are useful to analyze whether banks change behaviour and if this benefits the functioning of a network. Both the subprime crisis in 2007 and effects from Sept. 11th, 2001 had substantial influence on the network topology of the US financial market, cf. Soramäki et al. (2007) and Kroszner (2007). The Danish payments and money market networks were unaffected by the subprime crisis in a data set for the period July-September 2007. Pröpper et al. (2008) reach the same conclusion in a similar study for credit markets in the Netherlands.

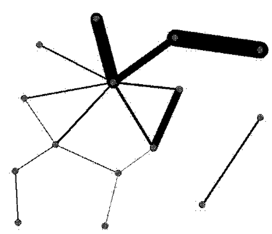
The operational disturbance implied a different structure of the networks during the day, cf. figure 8 and 9 and table 6. By the end of the day, almost all of the topological measures were significantly different from the 2006 average, cf. table 6. The activity and size of the payments network decreased significantly. The average path length had decreased significantly by the end of the day, whereas the connectivity and clustering of the payments network increased significantly. That is, the payments network became narrower.

In opposition to this, the activity in terms of volume and the size of the money market increased although the average value of each money market loan had decreased significantly by the end of the day. The connectivity of the money market network decreased significantly, whereas the average path length and average node degree increased. Thus, although the actual number of links out of the potential number of links decreased, the average number of links per active bank increased in the money market. All in all, the money market became wider during this event.

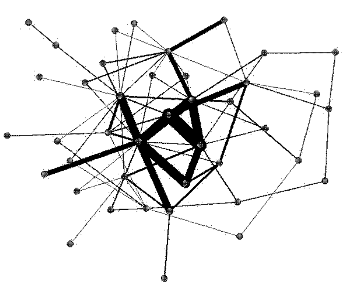
Figure 9: Operational disruption in the money market network



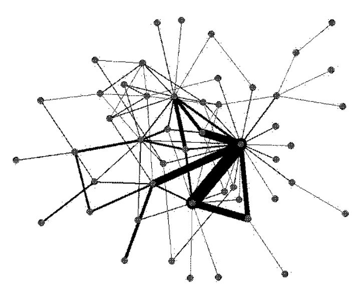
a: Normal day, 1:40 p.m.



b: Nov 16th, 1:40 p.m.



c: Normal day, End of day



d: Nov 16th, End of day

Note: The figures are weighted by value.

Table 6: Effects of an operational breakdown in the networks

	2006	Confidence limits		Operational breakdown	
	Average	Lower	Upper	End of day	1:40 p.m.
Payments					
Volume	2,162.4	2,127.1	2,197.6	**1,883.0**	**220.0**
Value, billion DKK	105.5	102.2	108.9	**80.6**	**6.7**
Nodes	59.1	58.4	59.8	**55.0**	**33.0**
Links	282.6	277.5	287.7	**260.0**	**70.0**
Connectivity, per cent	8.3	8.2	8.4	**8.8**	**6.6**
Clustering	0.53	0.53	0.54	**0.55**	**0.27**
Average path length	2.48	2.47	2.49	**2.46**	**1.75**
Average node degree	4.77	4.73	4.81	4.73	**2.12**
Money market					
Volume	86.4	83.2	89.7	**103.0**	**19.0**
Value, billion DKK	22.9	21.9	23.9	**20.9**	**3.5**
Nodes	27.4	26.6	28.3	**29.0**	**15.0**
Links	75.0	72.1	77.9	**82.0**	**16.0**
Connectivity, per cent	11.2	10.5	11.9	**10.1**	**7.6**
Clustering	0.17	0.16	0.18	0.18	**0.08**
Average path length	2.94	2.90	2.99	**3.01**	**0.54**
Average node degree	2.69	2.64	2.73	**2.83**	**1.07**

Note: Mean values of selected summary statistics for the core. Confidence limits for the 95 % confidence interval are used to determine the significant variables, which are bold. Clustering, average path length and average node degree are reported with 2 decimals.

The drop in payments network activity and boom in overnight money market loans are in opposition to the seasonal effects by the turn of the month, cf. table A4.

Although the operational disruption of the system had a large impact on the topologies of the payments and the money market networks, these effects were temporary. If the operational event had lasted longer, these effects might have been even more pronounced.

8.2. Payment disruption by a major participant

One of the largest commercial banks in Denmark, Danske Bank, was not able to send payments in the large-value payment system on two successive days in March 2003. This was caused by a major it-problem[18]. The Danish central bank supplied the banks with extra liquidity to overcome a potential lack of liquidity in the markets as the payment disruption as the major participant was able to receive, but could not send transactions to other banks.

The effects on the networks' structures were most pronounced on the first day of the crisis, Wednesday March 12th. The activity and size of the payments network decreased, whereas the activity in terms of volume and the size of the money market network increased by around 50 per cent on this day although the average size of an overnight money market loan decreased cf. table 7. Connectivity and clustering increased significantly in the payments network, whereas the average path length and the average node degree decreased. In the money market, the effects on these four variables were opposite.

[18] For a description of this event and how it was handled see Berlingske (2003a, 2003b).

Table 7: Effects of payment disruptions by a major participant

	Average March 2003	Confidence limits Lower	Upper	March 12	March 13
Payments					
Volume	2,306.5	2,267.4	2,345.6	**1,505.0**	**1,625.0**
Value, billion DKK	145.8	142.1	149.5	**119.7**	**128.6**
Nodes	56.5	55.9	57.2	**49.0**	**58.0**
Links	281.2	275.8	286.5	**227.0**	**254.0**
Connectivity, per cent	9.0	8.9	9.1	**9.7**	**7.7**
Clustering	0.50	0.50	0.51	**0.58**	**0.43**
Average path length	2.46	2.45	2.47	**2.44**	**2.57**
Average node degree	4.95	4.91	4.99	**4.63**	**4.38**
Money market					
Volume	64.4	61.3	67.5	**92.0**	**46.0**
Value, billion DKK	18.7	17.8	19.6	**8.8**	**7.8**
Nodes	24.1	23.0	25.1	**37.0**	**19.0**
Links	56.9	54.2	59.7	**87.0**	**44.0**
Connectivity, per cent	12.8	11.7	13.9	**6.5**	12.9
Clustering	0.17	0.16	0.18	**0.10**	0.17
Average path length	3.03	2.97	3.09	**3.70**	**2.96**
Average node degree	2.24	2.19	2.28	**2.35**	**2.32**

Note: The note to table 6 also applies here. The average of March 2003 excludes data from March 12th and March 13th.

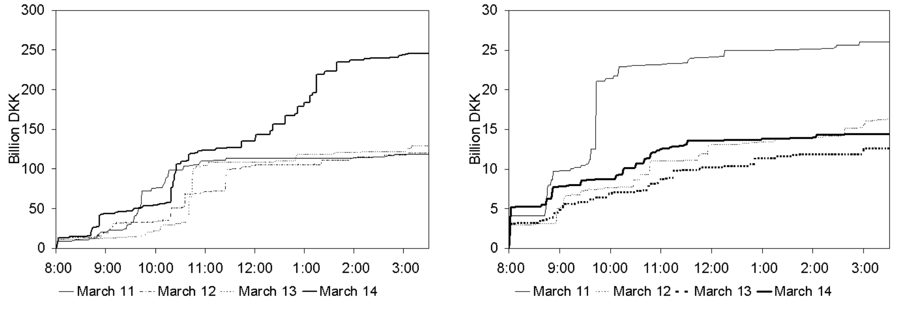
On the second day of this event, the major participant informed the public about the it-problem and its implications for the bank's business. Together with the significant boom in activity and size of the money market of the first day of the crisis, this lead to a decrease in activity and size of the money market network on the second day of the crisis. This decreased the average path length and average node degree.

The activity and the size in terms of links of the payments network remained significantly lower than the average for March 2003, but more banks became active in the payments network on the second day of the crisis. This is reflected in the significant drop in connectivity, clustering and average node degree. The average path length increased, i.e. transactions had to pass less links to reach the final recipient of a transaction.

The disruptions by a major participant also caused an accumulated settlement demand in the payments network and this lead to a sharp increase in the value settled within this network on the first normal business day after the event, cf. figure 10.

Compared with the operational breakdown, the effects of payment disruptions by a major participant are larger in both networks. The structural changes in the networks' topologies were temporary. And it seems as if the other banks took precautionary actions towards the disturbance and continued settlements as far as possible in both networks.

Figure 10: Large bank payment disruption



a: Payments b: Money market

Note: Amounts settled in the networks during the day on selected dates in March 2003. March 11th (March 14th) was the last (first) business day before (after) the crisis, while the crisis had effect on March 12th and 13th. The value of overnight loans in the money market increase by coincidence on March 11 as there are no holiday effects or effects of additional liquidity adjustments by the central bank this day. The opening time of the large value payment system was 8.00 a.m.-3.30 p.m. until June 1st 2003.

9. CONCLUSION

The topological analysis shows that the structure of the Danish money market is different from the structure of the payments network. This is a consequence of the difference in the nature of transactions in the networks. Transactions in the money market network are driven by banks' behaviour whereas transactions in the payments network arise from banks' proprietary transactions as well as customer driven transactions. In the payments network, two commercial banks are responsible for a rather large share of the total activity, whereas the banks in the core of the money market are of more equal size. Both networks are rather concentrated.

Our results show that the distribution of in- and out-degrees follow the exponential or the negative binomial distributions in the payments network, while the exponential distribution captures the distribution of in- and out-degrees quite well in the money market. In other countries, in- and out-degrees follow power-law distributions, but power-law distributions are clearly rejected in our data set.

We find clear evidence of seasonal effects for both networks. The results show that the payments network becomes wider by the turn of the month and quarter and on the first business day following a holiday. In contrast to this, weekday effects drive the calendar effects observed in the money market.

Event studies of an operational disruption imply a different structure of the networks during the day. Although the structure of the networks is almost normal by the end of the day, the daily activity of the payments network decreased considerably. In contrast to this, the daily activity of the money market increased. The topological effects of this event are in line with the seasonal effects by the turn of the month but with the opposite signs. The effects of the operational event were temporary, but might have been more pronounced in case the operational event had lasted longer than it did. The event study of payment disruptions by a major participant decreases the level of activities in both networks; especially on the first day of the event. An accumulated settlement demand was build up in the payments network, which was released on the first normal business day after the crisis leading to a sharp increase in the value settled in the payments network.

It could be interesting to see if the payments network builds up in a different way than the money market network during the day. At the moment a rather large fraction of the settlements take place before noon both in the money market and in the payments network, but the effects of a different timing of settlements on the structure of the networks is a question for further research.

ACKNOWLEDGMENTS

We thank Danmarks Nationalbank for providing us with the data set for the analysis. This paper has benefited from helpful comments from Enghin Atalay, Asger Lau Andersen, Bjarne Astrup Jensen and seminar participants in the Annual Workshop, Danish Graduate Programme in Economics, 2007; the Annual Workshop, The Danish Doctoral Educational Network in Finance, 2008; The Danish Economic Society's Annual Meeting, 2008; Danmarks Nationalbank, 2008 and Norges Bank, 2008.

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10. APPENDIX

10.1. More summary statistics

This section provides more summary statistics for the networks.

Table A1: More summary statistics, payments and money market networks, 2006

	Mean	Median	Min	Max	Std
Payments					
Distance measures					
Diameter	5.5	5.0	4.0	8.0	0.7
MDF, M(2)	54.6	54.8	44.7	65.4	4.2
MDF, M(3)	91.2	91.4	83.2	97.8	2.7
MDF, M(4)	99.1	99.3	94.9	100.0	0.8
MDF, M(5)	99.9	100.0	96.9	100.0	0.3
Degree distribution					
max k^{in}	34.6	34.0	24.0	51.0	4.4
max k^{out}	29.0	29.0	22.0	53.0	3.9
Centrality measures					
Betweenness, links	30.3	30.3	20.7	38.4	3.7
Betweenness, nodes	86.0	85.8	61.9	125.4	10.5
	Mean	Median	Min	Max	Std
Money market					
Distance measures					
Diameter	6.7	7.0	2.0	10.0	1.3
MDF, M(2)	42.1	40.6	26.6	100.0	9.5
MDF, M(3)	71.6	71.4	47.3	100.0	10.0
MDF, M(4)	89.7	91.0	61.6	100.0	7.3
MDF, M(5)	96.6	98.0	71.7	100.0	4.2
MDF, M(6)	98.9	99.9	82.0	100.0	2.3
MDF, M(7)	99.7	100.0	90.1	100.0	1.1
Degree distribution					
max k^{in}	10.3	10.0	2.0	24.0	3.6
max k^{out}	10.3	10.0	2.0	21.0	3.4
Centrality measures					
Betweenness, links	29.2	29.4	2.0	51.1	8.4
Betweenness, nodes	52.1	52.5	0.7	92.6	16.5

Note: The data reported refer to the core. The Mass Distribution Functions, MDF, are estimated based on outgoing payments from a node. max k^{in}(max k^{out}) is the maximum number of links ending in (starting from) a node.

10.2. Summary statistics of transactions network

This section contains the network topological measures of the transactions network, i.e. the network based on the full data set.

Table A2: Summary statistics, transactions network, 2006

	Mean	Median	Min	Max	Std
Basic network properties					
Volume	2,355.9	2,337.5	1,607.0	4,171.0	323.6
Value	131.7	128.9	46.9	224.8	30.8
Nodes	67.8	67.0	57.0	88.0	4.8
Links	373.7	368.0	283.0	713.0	48.4
Connectivity, per cent	8.3	8.2	6.9	10.1	0.7
Reciprocity, per cent	24.0	24.0	19.1	28.4	1.9
Clustering	0.5	0.5	0.4	0.6	0.0
Average path length	2.4	2.4	2.2	2.6	0.1
Average node degree, k	5.5	5.5	4.6	8.1	0.4
Link weight, value	0.4	0.4	0.1	3.4	0.5
Link weight, volume	6.3	6.3	4.3	8.8	0.6
Node strength, value	1.9	1.9	0.8	3.4	0.5
Node strength, volume	34.7	34.4	23.0	48.8	3.9
Distance measures					
Diameter	5.1	5.0	4.0	8.0	0.6
MDF, M(2)	58.1	58.1	48.7	69.8	4.2
MDF, M(3)	94.3	94.4	87.1	98.8	2.1
MDF, M(4)	99.7	99.8	96.4	100.0	0.5
MDF, M(5)	100.0	100.0	98.1	100.0	0.1
Degree distribution					
max k^{in}	40.9	40.0	31.0	57.0	4.7
max k^{out}	34.5	34.0	24.0	54.0	4.2
Centrality measures					
Betweenness, links	29.4	29.0	22.5	38.0	3.2
Betweenness, nodes	93.9	93.5	74.3	121.0	9.1

Note: The notes to table 4 and table A1 also apply to this table.

10.3. Seasonal effects

This section contains the regression results regarding seasonal effects in the networks.

Table A3: Seasonal effects, payments network, 2006

		Holidays in		End of		Liquidity adjustment		Weekdays				R^2
	Intercept	Denmark	US	Quarter	Month	Expected	Unexpected	Monday	Tuesday	Thursday	Friday	
Nodes	59.9**	5.7*	1.1	17.1**	6.1**	2.5*	1.7	-3.6**	-1.4	-1.9*	2.8*	31.9
	0.8	2.4	1.4	2.2	1.2	1.3	1.5	1.1	1.1	1.1	1.3	
Links	279.2**	73.9**	18.5*	146.3**	48.5**	18.1**	24.6*	-13.2*	-13.4*	-5.2	38.7**	40.7
	4.8	15.4	6.9	10.0	7.0	7.1	10.6	6.9	5.6	6.4	11.4	
Value, billion DKK	95.1**	23.8**	15.9*	43.2**	22.0**	5.4	0.5	6.1	-2.5	0.7	22.9**	20.8
	3.2	8.9	6.9	8.8	6.8	6.1	7.9	6.4	4.8	5.0	4.8	
Volume, thousands	2.2**	0.6*	0.4**	0.7**	0.2*	0.2**	0.1	-0.2**	-0.2**	-0.1*	0.0	26.7
	0.0	0.3	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	
Average node degree	4.7**	0.7**	0.2**	0.9**	0.3**	0.1*	0.3**	0.1	-0.1*	0.1	0.4**	34.6
	0.0	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.0	0.1	0.1	
Connectivity, per cent	8.0**	0.4	0.2	-0.8*	-0.3	-0.2	0.2	0.6**	0.0	0.4*	0.2	9.5
	0.1	0.5	0.3	0.4	0.2	0.2	0.2	0.2	0.2	0.2	0.2	
Average path length	2.50**	-0.08	-0.05*	-0.01	-0.01	0.00	-0.04**	-0.03*	0.03*	-0.02	0.00	7.8
	0.01	0.06	0.02	0.03	0.02	0.02	0.01	0.02	0.02	0.02	0.01	
Clustering, per cent	53.7**	2.6	2.7	-1.1	0.3	1.7*	1.7	0.3	-0.2	0.8	-4.5**	17.2
	0.6	2.3	1.9	2.3	1.2	0.9	2.3	1.1	0.9	0.9	0.9	

Note: For each explanatory variable, the first line of results is parameter estimates and the second robust standard errors. Significant parameters on a 1 (5) per cent level is marked with ** (*) in a one-tailed t-test (df=200). Average path length is reported with two decimals. Estimations are based on data for the core of the network. Holiday effects are measured on the first business day following a closing day. American holidays are holidays in addition to Danish holidays. European holidays are captured by the dummies for Danish holidays and turn of month. The turn of month (quarter) includes the first and the last opening day in each month (quarter). The Danish central bank adjusts liquidity in addition to the regular adjustments on Fridays on days with ingoing tax payments or outgoing social benefits etc. Additional adjustments will normally be announced in advance (expected adjustments), but a few adjustments are not announced.

Table A4: Seasonal effects, money market network, 2006

		Holidays in		End of		Liquidity adjustment		Weekdays				R^2
	Intercept	Denmark	US	Quarter	Month	Expected	Unexpected	Monday	Tuesday	Thursday	Friday	
Nodes	30.6**	-4.7	1.5	-2.1	-5.0**	-0.9	3.1	-5.8**	-3.6**	0.5	-5.6**	22.6
	0.8	4.2	1.5	1.8	1.3	1.3	3.5	1.2	1.2	1.1	1.2	
Links	84.1**	-12.4	5.3	2.5	-14.9**	0.1	14.8	-18.9**	-9.6*	1.8	-17.4**	18.6
	2.9	13.3	6.1	6.9	4.0	4.7	12.0	4.2	4.2	4.1	4.0	
Value, billion DKK	24.4**	-3.4	2.8	-3.6	2.2	-1.0	-1.7	-4.2**	-1.7	1.1	-2.7	7.8
	1.1	4.3	3.7	2.2	2.4	1.4	2.1	1.5	1.6	1.5	1.5	
Volume, thousands	0.1**	-0.0	0.0	0.0	-0.0**	0.0	0.0	-0.0**	-0.0*	0.0	-0.0**	17.6
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
Average node degree	2.7**	-0.3	0.1	0.4**	-0.1	0.1	0.2*	-0.1*	-0.0	0.0	-0.1	8.6
	0.0	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	
Connectivity, per cent	9.2**	10.6	-1.2	2.3*	4.1*	0.4	0.7	2.8**	2.3*	-0.0	2.3**	17.3
	0.4	7.6	1.3	1.1	2.5	0.7	2.0	1.0	1.4	0.5	0.7	
Average path length	3.05**	-0.35	-0.11	-0.16	-0.12	-0.09	-0.16**	-0.10	-0.12	-0.03	-0.12*	6.9
	0.05	0.26	0.10	0.11	0.13	0.07	0.05	0.07	0.07	0.06	0.06	
Clustering, per cent	15.4**	-2.6	3.8	1.4	0.3	2.4	2.1	1.1	1.7	2.2	2.4	2.8
	1.1	3.3	3.7	2.1	2.3	2.0	2.4	1.6	1.6	1.7	1.6	

Note: The note to table A4 also applies to this table.

10.4. The Furfine algorithm

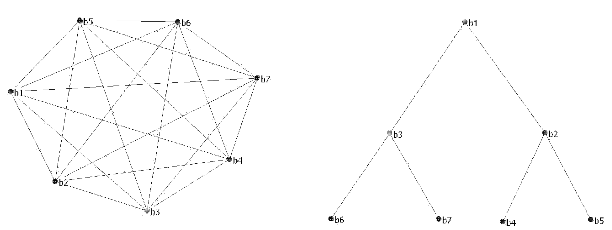
The Furfine algorithm is used to identify overnight money market loans in order to split our data set into transactions stemming from two economically different networks.

The algorithm defines a transaction as an overnight money market loan if 1) the borrowed amount is at least 1 million DKK in integer numbers, 2) the borrowed amount is repaid with interest the next business day and 3) the interest amount is within an acceptable range, $i = [i_{Low}, i_{High}]$. The lower (upper) bound of this interval is the minimum interest rates on unsecured overnight lending reported by a panel of Danish banks minus (plus) 25 basis points. The acceptance range is extended with ± 25 basis points since "interest rates charged are likely to vary across transactions", cf. Furfine (1999, p. 26). The acceptance range on Danish data is smaller than the ± 50 basis points Furfine (1999) uses on Fedwire transactions. But broadening (decreasing) the acceptance range to ± 50 basis points (± 0 basis points) on Danish data gives almost the same classification of unsecured overnight lending by the algorithm.

10.5. Stylized networks

Two different extremes of stylized networks are illustrated in figure A2. In a complete network, a bank has links to all other banks in the network such that each bank submits and receives transactions to/from all other banks within the network. In a tree network bank 1 submits transactions to bank 2 and 3, which submits transactions to bank 4 and 5 respectively bank 6-7. Another type is random networks, which is constructed by adding links at random to a given set of nodes. Stylized networks are not observed empirically but they are useful as benchmarks for analytical purposes.

Figure A1: Stylized networks



a: Complete network

b: Tree network

10.6. Statistical measures used

The list below gives a short description of the statistical measures used above. These measures are common in the network topological approach.

Average node degree, which is a measure of the average number of links per node.

Average Path Length. The average path length measures the average number of links connecting two nodes in a network via the shortest possible path, i.e. this is a measure of the number of links a transaction must pass to reach another bank in the network. The average path length can be estimated using payments received in or submitted from a node.

Betweenness (for nodes or links). A centrality measure, which measuress the number of paths between other nodes that run through node i. The more paths that go through node i, the more central is this node.

Clustering. Measures to which degree two banks, which is both linked up with bank x, is also linked up with each other. The more links the banks, which bank x has linked up to, have, the more banks does bank x have access to. Clustering varies between 0 (tree network) and 1 (complete network). In a random network, i.e. where the links between banks are distributed randomly, the clustering coefficient is equal to the connectivity of the network. Clustering can be estimated both using the payments send to a node and payments submitted from a node.

Connectivity. The share of actual links out of potential links (per cent). The connectivity varies between $\frac{1}{nodes}$ (tree network) and 1 (complete network).

Diameter. The maximum distance between two nodes in a network.

Link weight. This is a measure of the importance of the links, when the links are corrected for how many transactions (or value of transactions) they handle. That is, a link, which handles 10 transactions, is more important than a link, which handles 1 transaction and vice versa for links weighted by values transferred.

MDF(x), Mass Distribution Function, where x is the distance from a node. That is, MDF(2) says how large a share of all the nodes in the network, which can be reached within the distance 2 from a node. The mass distribution function can be estimated using in- or outgoing payments to/from the nodes in a network.

Maximum in-degree of a node (max k^{in}). This is a measure of the maximum number of links that terminate in a node.

Maximum out-degree of a node (max k^{out}). This is a measure of the maximum number of links that originates from a node.

Node strength. This is a measure of the importance of the nodes weighted by the number (of value) of transactions through each node. That is, a node, which handles 50 transactions, is more important than a node, which handles 10 transactions and vice versa for nodes weighted by values handled.

Reciprocity. Reciprocity measures the share of links for which there is a link in the opposite direction (per cent). Varies between 0 (tree network) and 1 (complete network).